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The number of different cyclic sets, or different periods of the same length, for each denominator n is equal to $\phi(n)$ divided by the exponent to which 10 belongs, modulus n. This is true since $\phi(n)$ gives the number of fractions having numerators in the multiplication group G and the exponent to which 10 belongs, modulus n, gives the number of these fractions in each cyclic set. For instance, the number 10 belongs to exponent 6, modulus 21, and $\phi(21)$ is 12. Therefore, there are two different cyclic sets for $n=21$, as was seen in the previous example.

The operation of finding the numbers composing the different cyclic sets of the $\phi(n)$ fractions is very much shortened by the fact, that, in general, the different cyclic sets, or periods, occur as complements of each other. Since -1 and $+1$ always occur among the $\phi(n)$ numbers of G, the $\phi(n)$ fractions whose numerators differ only in respect to sign, modulus n, have periods occurring in complementary pairs. Then if a period has been obtained, its complementary period is found by subtracting each digit of the first period from nine. If 10 is a primitive root of n, then -1 is in the cyclic sub-group G_1. The period is in this case of even length and the second half of the period is obtained by subtracting the digits of the first half from nine. The period is of even length since G_1 then contains an operator of order two and the order of the operator must divide the order of G_1. If -1 is not in G_1, then the index of G_1 under G must be even and the periods are complementary.

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THE DEFLECTING FORCE OF THE EARTH'S ROTATION AND FOUCAULT'S PENDULUM: AN ELEMENTARY ANALYSIS.

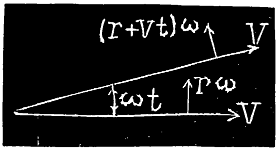
By **W. H. JACKSON**, Haverford College, Haverford, Pa.

The intrinsic interest in tangible evidences of the Earth's rotation makes it desirable to introduce the following results at as early a stage as is possible. This is the justification for the following elementary exposition.

1. *General Method.* Let a point move about a center with constant angular velocity ω, and recede from it radially with constant velocity v, and let it be initially at a distance r from the center.

After a time t the radial velocity is in a direction making an angle ωt with its initial direction. The component velocities along and perpendicular to the initial radius CP are therefore initially v, $r\omega$. After a time t they are, respectively,

$$v \cos \omega t - (r + v t)\omega \sin \omega t,$$
$$v \sin \omega t + (r + v t)\omega \cos \omega t.$$



To find the components of acceleration, we must divide the component changes in velocity by t and proceed to the limit when this is small. Remembering that

$$\lim_{t \doteq 0}\left(\frac{1-\cos\omega t}{t}\right)=0,\quad \lim_{t \doteq 0}\left(\frac{\sin\omega t}{t}\right)=\omega,$$

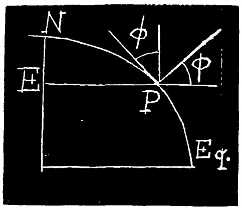
we obtain along the radius and perpendicular to it, respectively,

$$-r\,\omega^2,\quad 2\,v\,\omega. \tag{1}$$

A body moving with uniform velocity relative to the Earth would have on that account a certain acceleration relative to "fixed axes." This acceleration reversed will therefore be observed when the body moves freely on the Earth's surface.

The proof does not assume that the Earth is spherical. The latitude of any place is defined as the angle which the apparent direction of gravity makes with the plane of the equator. The latitude of a place is positive or negative according as its position lies to the north or south of this plane.

We shall consider the effect of motion along standard directions, and afterwards combine the results.



2. North-South Motion. Suppose that a body is moving with uniform velocity v from south to north. When at rest in a place P, of latitude ϕ, it has a uniform angular velocity ω about E, the projection of P on the Earth's axis. A velocity v, due north has a component $-v\sin\phi$ along EP, and the corresponding change in acceleration produced by this velocity is

$$-2\,\omega\,v\sin\phi \text{ by (1)},$$

in the direction of the Earth's rotation from west to east. Relative to the Earth, a body moving freely would experience an acceleration

$$2\,\omega\,v\sin\phi \text{ to the east.} \tag{2}$$

3. West-East Motion. Suppose that a body is moving with uniform velocity v from west to east. The only change in acceleration is due to the increase in angular velocity from ω to $(\omega+v/PE)$; it is equal to

$$-PE\left(\omega+\frac{v}{PE}\right)^2+PE.\omega^2 \text{ by (1)},$$

$$=-2\,\omega\,v\left(1+\frac{v}{2\,\omega\,PE}\right) \text{ along } EP. \tag{3}$$

If we regard the ratio which the velocity of the body relative to the Earth bears to that due to the Earth's rotation as negligible, we obtain a horizontal component of acceleration of the same magnitude as before,

$$2\,\omega\, v \sin\phi \text{ to the south.} \tag{4}$$

4. Any motion parallel to the Earth's surface. Any motion parallel to the Earth's surface may be compounded of a south-north and west-east motion. A body moving freely in any direction with velocity v on the Earth's surface will therefore possess an acceleration

$$2\,\omega\, v \sin\phi \tag{5}$$

in a direction obtained by a clockwise rotation of a right angle from its direction of motion.

5. Foucault's Pendulum. Suppose the bob of a simple pendulum is started so as to oscillate in any vertical plane, if r denotes its distance from its equilibrium position and v its velocity along that radius, the relative acceleration due to the Earth's rotation is

$$2\,\omega\, v \sin\phi.$$

But by (1), this is just the acceleration due to a rotation of angular velocity ($\omega \sin\phi$) about the vertical axis. The corresponding radial acceleratiod of $-r\,\omega^2 \sin^2\phi$ is absent but that is negligible owing to the smallness of ω. That is, a pendulum started swinging in any vertical plane will rotate in the clockwise direction with uniform angular velocity

$$\omega \sin\phi. \tag{6}$$

6. The deflection of a body falling vertically. Suppose a body to move with uniform velocity v along EP, the change in its real acceleration is

$$2\,\omega\, v \text{ to the east by (1).}$$

A body moving freely along EP, therefore possesses a relative acceleration of

$$-2\,\omega\, v \text{ to the east.}$$

If a body falls freely with velocity v, this velocity may be resolved into

$$-v \sec \phi \text{ along } EP,$$

and

$$-v \tan \phi \text{ to the north.}$$

Its resulting acceleration to the east is therefore

$$2\,\omega\, v(\sec \phi - \tan \phi \sin \phi).$$

That is,

$$2\,\omega\, v \cos \phi \text{ to the east.} \tag{7}$$

The total displacement in a fall from a height h is therefore given approximately by writing $v=gt$, integrating with respect to t twice and substituting $t=(2\,h/g)^{\frac{1}{2}}$. The displacement is found to be

$$\tfrac{1}{3}\,\omega \cos \phi (8\,h^3/g^3)^{\frac{1}{2}}. \tag{8}$$

Finally, there is one component of relative acceleration which has not been written down; that is the vertical component due to the velocity v from west to east. On reference to equation (3), it is seen that this component is

$$2\,\omega\, v \cos \phi \text{ vertically upwards.} \tag{9}$$

7. The general equations of relative motion. If we take axes x, y, and z to be east, north, and vertically upwards, respectively, the preceding results are expressed by the usual equations for the motion of a body moving freely:

$$\begin{aligned} x'' &= 2\,y'\,\omega \sin \phi - 2\,z'\,\omega \cos \phi, && \text{by (2), (7).} \\ y'' &= -2\,x'\,\omega \sin \phi, && \text{by (4).} \\ z'' &= 2\,x'\,\omega \cos \phi, && \text{by (9).} \end{aligned}$$

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ON THE REPRESENTATION OF NUMBERS AS THE SUM OF TWO SQUARES.

By M. KABA in Collaboration with L. E. DICKSON.

Consider the representations expressible as the sum of the squares of two numbers. Following Jacobi's notation for the theta-function with a special argument, we have

$$\theta(k) = \sqrt{\frac{2\,k}{\pi}} = 1 + 2q + 2q^4 + 2q^9 + \ldots$$